## Section 2.5

- **The Chain Rule:** If $f$ and $g$ are differentiable functions, then the composite function $m(x) = f(g(x))$ is differentiable and is given by

$$m'(x) = f'(g(x)) \cdot g'(x)$$

- **Specific Cases of the Chain Rule:**
  - If $y = (g(x))^n$ then $y' = n(g(x))^{n-1} \cdot g'(x)$
  - If $y = e^{g(x)}$ then $y' = e^{g(x)} \cdot g'(x)$
  - If $y = b^{g(x)}$ then $y' = \ln b \cdot b^{g(x)} \cdot g'(x)$
  - If $y = \ln(g(x))$ then $y' = \dfrac{1}{g(x)} \cdot g'(x)$
  - If $y = \log_b(g(x))$ then $y' = \dfrac{1}{\ln b} \cdot \dfrac{1}{g(x)} \cdot g'(x)$

**On Problems 1-9, find the derivative of the function.**

1. $f(x) = \left(7x^2 + 9x + 4\right)^{10}$

2. $g(x) = 5e^{-5x^2}$

3. $h(x) = \log_7\left(2x^4 - 3x + e^x\right)$

4. $g(x) = 4x\left(2^x + \sqrt[5]{x^2} + \dfrac{5}{x}\right)^9$

5. $f(t) = \dfrac{3}{\sqrt[5]{7t^3 + 2^t}}$

6. $k(x) = \left(\dfrac{7x}{3x^3 - 4}\right)^8$

7. $L(x) = 3x^9 \cdot 9^{(2x^7+4x)}$

8. $m(t) = (\log_8(5 + 4e^t))^9$

9. $C(t) = \ln\left(\dfrac{4(3t-7)^3\sqrt[6]{4t+7}}{5t^2-4}\right)$

10. If $f(x)$ is a differentiable function with $f(0) = 1$ and $f\,'(0) = 2$, what is $g\,'(0)$ if $g(x) = \dfrac{(4f(x)+x^2)^8}{e^x}$?

11. Find the equation of the line tangent to the curve of $f(x) = \sqrt[9]{32x^2} + \ln\left[(x-3)^3\right]$ at $x = 4$.

12. Anna has a bank account that earns interest at a rate of 3.4% per year compounded continuously. If she placed \$3,000 into the account when she opened it, at what rate (in dollars per year) is the account growing after 10 years?

13. The profit function for a company that sells water flossers is given by $P(x) = 10\sqrt{x^2 - 1} - 200$, when $x$ water flossers are sold. Find (and interpret) the marginal profit (in dollars per water flosser) when 50 water flossers are sold.

## Section 2.6 Part 1 - Implicit Differentiation

- Sometimes we want to find the derivative of a function but cannot easily solve for $y$. In these situations we can use **implicit differentiation** to find the derivative $\left(y\,' = \dfrac{dy}{dx}\right)$ by following these steps:
  - Take the derivative with respect to the independent variable (typically $x$) of both sides. Use the Chain Rule when necessary.
  - Move all terms that have $\dfrac{dy}{dx}$ in it to the left-hand side and all terms that do not have $\dfrac{dy}{dx}$ in it to the right-hand side.
  - Factor $\dfrac{dy}{dx}$ out of all terms on the left-hand side and solve for $\dfrac{dy}{dx}$.

**For problems 14-17, use implicit differentiation to find $\frac{dy}{dx}$.**

14. $7x - 14e^x + \sqrt[3]{y} = y - 2x^2 + 9$

15. $5e^{2x} - 4\sqrt{y} = 3x^2 - 5^y$

16. $3xe^y - 7x^2y^3 = 10$

17. $\dfrac{3x^2 - 4y}{e^y + 7} = x$

18. For the equation given, evaluate $\frac{dy}{dx}$ at the point $(1, 0)$.

    $y = \ln\left(10x^3 - 4y^5\right)$

19. Find the equation of the line tangent to the curve of $\sqrt{x} - \sqrt{y} = 1$ at the point $(9, 4)$